TOWER OF HANOI

by hourglasslearning.com

Description of the game

Initially there is a stack of rings on a rod in order of increasing size from top to bottom. There are two other empty rods. The goal is to transfer the stack of rings to one of the empty rods following these three rules:

1. Only one ring can be moved each time.
2. In each move, take only the top ring on one of the rods and move it to another rod.
3. At no point in time, a larger ring may be placed on top of a smaller ring.

A **B** **C**

You can play the game with any number of rings. It may be a good idea to start with a small number of rings and increase gradually once you feel comfortable with the game. Keep track of the number of moves and fill in the following table. The goal is to keep the number of moves *as few as possible.*

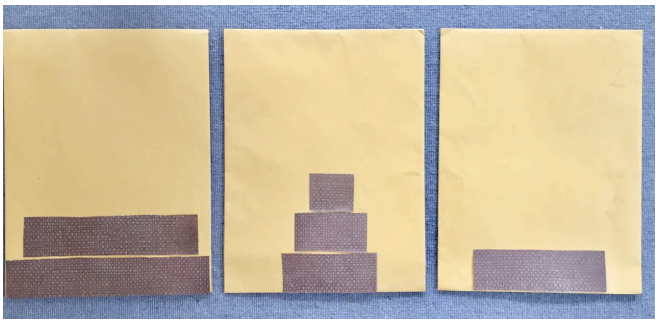
Number of rings	Number of moves
1	
2	
3	
4	
5	
6	
7	

HOW TO CONSTRUCT THE TOY?

The Melissa & Dough Geometric Stacker , which is a popular toy for toddlers, is also a great toy for the Tower of Hanoi.

Another common toy that many families have for the toddler years is the stacking rings toy. Add a metal chair to that and you'll get a very stable construction to play the Tower of Hanoi.

If you don't have any premade toys at home, it is a good idea to ask the kids to construct the toy themselves. It builds up excitement for them. One easy way to make it is to cut out strips of papers of different lengths as shown below. They may also color each strip with a different color.



Tower of Hanoi made from paper. A game in progress.

HOW TO PLAY THE GAME

Start with a small number of rings, for example 3. First, make sure the kids get a hang of the game and correctly follow the rules.

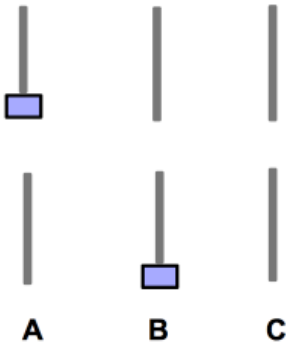
Let them record the number of moves they need to perform to complete the task on the table on page 1 and compare with the table below.

Number of rings	Minimum number of moves
1	1
2	3
3	7
4	15
5	31
6	63
7	127

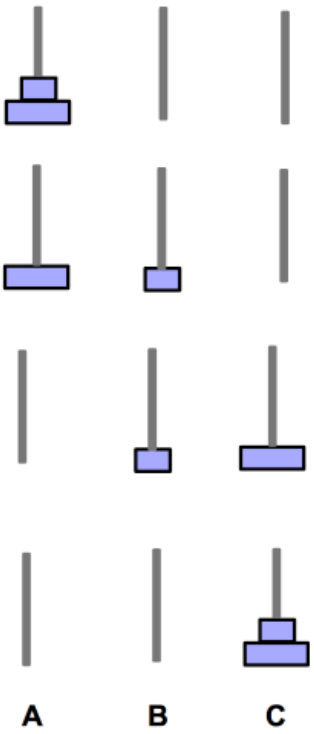
It takes time to achieve these minimum numbers of moves. The kids will have to find out a strategy to do so. A new player usually has redundant moves. Don't rush! Let them try on a different day. If you feel that the kids are not ready yet for a larger number of rings, try a few months later. The joy is in the discovery, not in being told the solution.

See next page for the solution.

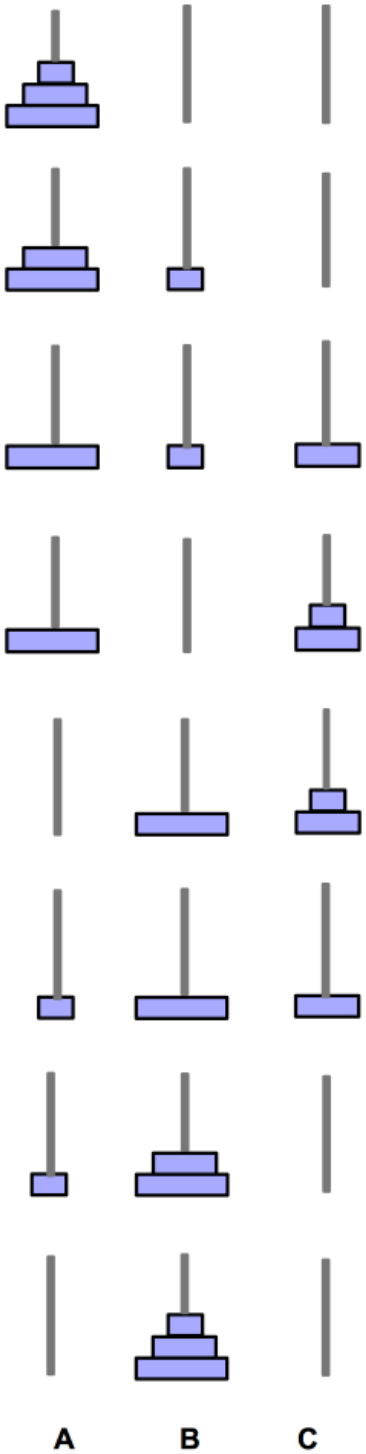
1 ring : 1 move



2 rings: 3 moves



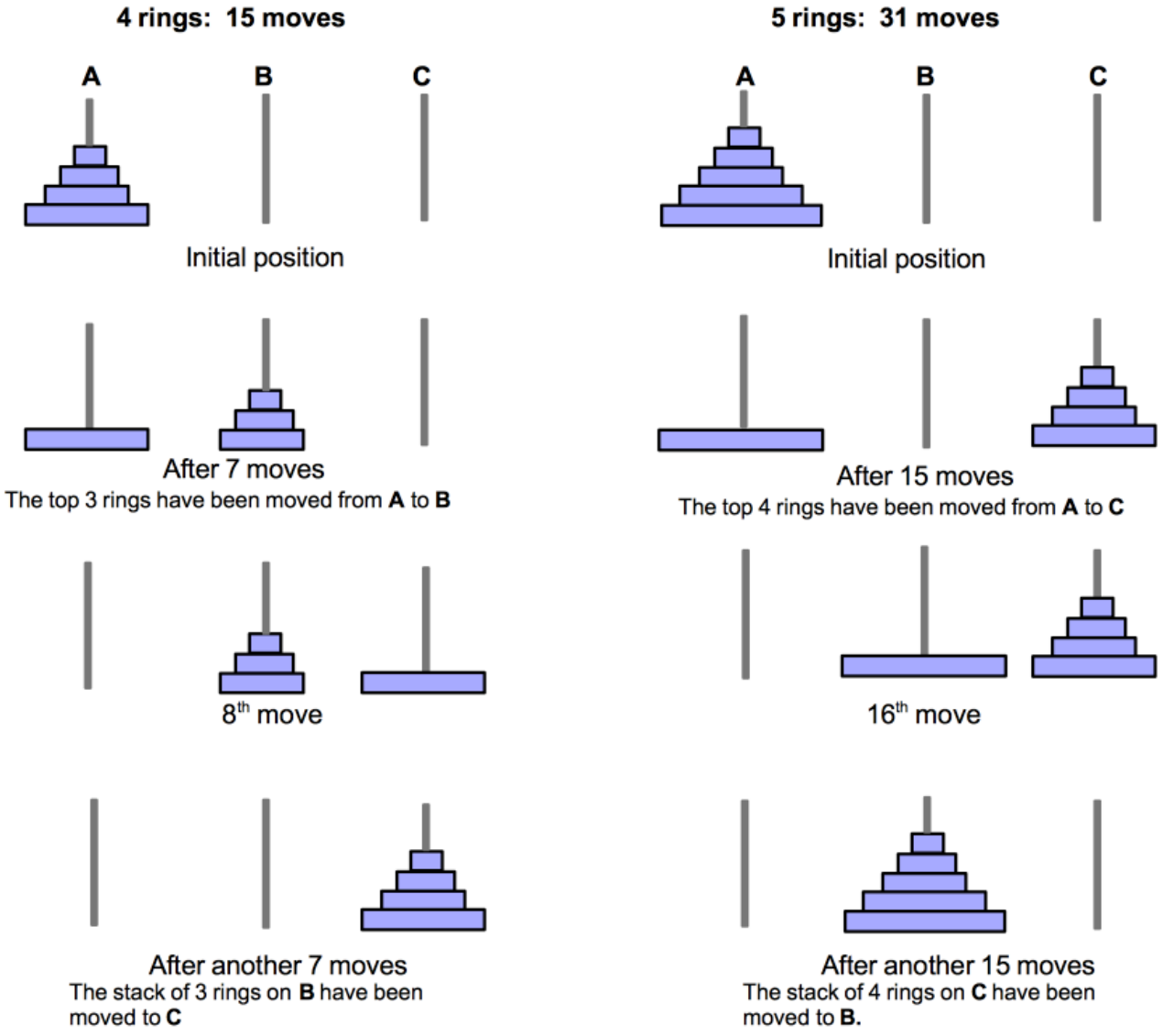
3 rings: 7 moves



The first 3 rings are pretty straightforward. When playing the game with more rings, the key to avoid having redundant moves is to move the top rings first, because we already know how to do that efficiently.

For example, to play the 4-ring game, first leave the largest ring untouched and move only the top 3 rings to one spare rod. We already know how to do that and it takes 7 moves. Next, move the largest ring to the other spare rod. This takes 1 move. Finally, move the the stack of 3 rings to the rod that is now holding the largest ring. Again this takes 7 moves. So in total, we need 7+1+7=15 moves.

To play the 5-ring game, move the top 4 rings first, and so on, so forth.



Recursion and the Russian dolls

The solution for the Tower of Hanoi game presented above has a very special feature: to move a stack of rings involves moving stacks of smaller numbers of rings using *exactly the same procedure.* For example, when faced with the task of moving 10 rings, we immediately recognize that we need to move the first 9 rings first, then in turn, in order to move 9 rings, we need to move the top 8 rings first, and so on so forth until there is only 1 ring when we know exactly what to do.

Kids who can move up to 5 or 6 rings using a minimum number of moves for sure have figured out this rule. Encourage them to explain the pattern in their solution in their own words. When you feel that it is the right time to explain the concept of recursion, one good analogy to use is the Russian doll-Matryoshka. Each doll actually consists of a number of nesting dolls. You open the doll only to find another one that looks exactly the same, just a bit smaller inside. Then you open the second doll again only to find another one that looks exactly the same, but a bit smaller inside. And that pattern continues until there is a solid tiny little doll (just like the 1-ring Tower of Hanoi). It is fascinating for a toddler to see the dolls the first time. Likewise, it is equally fascinating to discover the recursive feature of the solution of the Tower of Hanoi game.

Finally, a few words about the formula to calculate the minimum number of moves. If we have n rings, the minimum number of moves is 2^n-1. This fact can be proved by mathematical induction, but we can wait for now. At this age, it is more important for the kids to build mathematical intuition. If they can figure out the rules themselves and understand, or just feel the concept of recursion, they've already got a great intuition. And that is why this tower of Hanoi game is a treasure; it is a great game for learning and teaching for every one all the way from 7 year old to a computer scientist.